# Answers to Coursebook questions – Chapter 7.2

**1** **a** A black body is any body at absolute temperature *T* whose radiated power per unit area is given by $\sigma T^4$. A black body appears black when its temperature is very low. It absorbs all the radiation incident on it and reflects none.

**b** A piece of charcoal is a good approximation to a black body, as is the opening of a soft drink can.

**c** It increases by $\left(\frac{273+100}{273+50}\right)^4 = 1.8$.

**2** **a** The wavelength at the peak of the graph is determined by temperature and since the wavelength is the same so is the temperature.

**b** The ratio of the intensities at the peak is about $\frac{1.1}{1.9} \approx 0.6$.

**3** We have that $e\sigma AT^4 = P \Rightarrow T = \sqrt[4]{\frac{P}{e\sigma A}}$

i.e. $T = \sqrt[4]{\frac{1.35\times10^9}{0.800\times5.67\times10^{-8}\times5.00\times10^6}} = 278\ \text{K}$.

**4** **a** We must have that $\sigma AT^4 \propto \frac{1}{d^2} \Rightarrow T \propto \frac{1}{\sqrt{d}}$.

**b** Using our knowledge of propagation of uncertainties, we deduce that $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta d}{d}$ and so $\frac{\Delta T}{T} = \frac{1}{2}\times1.0\% = 0.005$.
Hence, $\Delta T = 0.005T = 0.005\times288 = 1.4\ \text{K}$.

**5** **a** Intensity is the power received per unit area from a source of radiation.

**b** $P = e\sigma AT^4$. $P = 0.90\times5.67\times10^{-8}\times1.60\times(273+37)^4 = 754\ \text{W}$.
Assuming uniform radiation in all directions, the intensity is then
$I = \frac{P}{4\pi d^2} = \frac{754}{4\pi(5.0)^2} = 2.4\ \text{W m}^{-2}$.

**6** **a** Imagine a sphere centred at the source of radius *d*.
The power *P* radiated by the source is distributed over the surface area *A* of this imaginary sphere.
The power per unit area, i.e. the intensity, is thus $I = \frac{P}{A} = \frac{P}{4\pi d^2}$.

**b** We have assumed that the radiation is uniform in all directions.

**7** **a** The peak wavelength is approximately $\lambda_0 = 0.65 \times 10^{-5}$ m , and so from Wien's law, $\lambda_0 T = 2.9 \times 10^{-3}$ K m , we find $T = \frac{2.9 \times 10^{-3}}{0.65 \times 10^{-5}} = 450$ K .

**b** The curve would be similar in shape but taller, and the peak would be shifted to the left.

**8** **a** Albedo is the ratio of the reflected intensity to the incident intensity on a surface.

**b** The albedo of a planet depends on factors such as cloud cover in the atmosphere, amount of ice on the surface, amount of water on the surface and colour and nature of the soil.

**9** **a** The energy flow diagram is similar to that in **Figure 2.4** (see page 438 in *Physics for the IB Diploma*).

**b** The reflected intensity is $350 - 250 = 100$ W m$^{-2}$ and so the albedo is $\frac{100}{350} = 0.29$ .

**c** It has to be equal to that absorbed, i.e. $250$ W m$^{-2}$ .

**d** Use $e\sigma T^4 = I \Rightarrow T = \sqrt[4]{\frac{I}{e\sigma A}}$ ,

i.e. (assuming a black body) $T = \sqrt[4]{\frac{250}{5.67 \times 10^{-8}}} = 258$ K .

You must be careful with these calculations in the exam. You must be sure as to whether the question wants you to assume a black body or not. Strictly speaking, in a model without an atmosphere the earth surface cannot be taken to be a black body – if it were, no radiation would be reflected!

**10** **a** The power incident on the lake is $P = mc\frac{\Delta\theta}{\Delta t} = (\rho A d)c\frac{\Delta\theta}{\Delta t} \Rightarrow \frac{P}{A} = \rho d c\frac{\Delta\theta}{\Delta t}$ , where $\rho$ is the density of water.

Then $\frac{\Delta\theta}{\Delta t} = \frac{I}{\rho d c} = \frac{340}{10^3 \times 50 \times 4200} = 1.6 \times 10^{-6}$ K s$^{-1}$. So to increase the temperature by 1 K the time required is $\frac{1}{1.6 \times 10^{-6}} = 6.2 \times 10^6 \text{ s} \approx 172$ hr .

**b** The heat capacity is $C = mc$ , where *m* is the mass of water. This can be estimated as $m = (\rho A d)$ , where *A* is the surface area of the water-covered area of the earth, *d* the average depth of water that can participate in thermal interactions (taken to be 300 m in this problem) and $\rho$ is the density of water. Assuming a water-covered area of 75% of the entire earth surface, we have that $A = 0.75 \times 4\pi R^2 = 0.75 \times 4\pi(6.4 \times 10^6)^2 = 3.9 \times 10^{14}$ m$^2$ . Then,

$$C = 10^3 \times 3.9 \times 10^{14} \times 300 \times 4200 = 5 \times 10^{23} \text{ J K}^{-1}.$$

**c** From $P = C\dfrac{\Delta\theta}{\Delta t}$ we find $340\times 3.9\times 10^{14} = 5\times 10^{23}\dfrac{1}{\Delta t} \Rightarrow \Delta t \approx 4\times 10^{6}\ \mathrm{s}$.

**11 a** For Venus, $I = \dfrac{3.9\times 10^{26}}{4\pi d^2} = \dfrac{3.9\times 10^{26}}{4\pi(1.08\times 10^{11})^2} = 2661\ \mathrm{W\ m^{-2}}$

and for Mars $I = \dfrac{3.9\times 10^{26}}{4\pi d^2} = \dfrac{3.9\times 10^{26}}{4\pi(2.28\times 10^{11})^2} = 597\ \mathrm{W\ m^{-2}}$.

Hence, repeating the calculation in **example Q3**,

$T_V = \sqrt[4]{\dfrac{2661}{4\times 5.67\times 10^{-8}}} = 329\ \mathrm{K}$ and $T_M = \sqrt[4]{\dfrac{597}{4\times 5.67\times 10^{-8}}} = 227\ \mathrm{K}$.

**b** This simple calculation, which ignores the greenhouse effect, gives a reasonable estimate for the temperature of Mars, implying that the greenhouse effect is not significant for Mars. For Venus, however, the estimate is completely wrong, indicating a very significant greenhouse effect for this planet.

**12** To begin with, $I = \dfrac{S}{4}$.

**a** In obvious notation according to the diagram let $I_1, I_2, I_3$ be the unknown intensities. The easiest one to find is $I_3$.

From the overall balance of the entire earth it follows that $I_3 = (1-\alpha)\dfrac{S}{4}$.

But this is the fraction of the earth radiated intensity that escapes, and so $I_3 = tI_1$.

This gives the value of $I_1$, $I_1 = \dfrac{(1-\alpha)}{t}\dfrac{S}{4}$.

Now look at the energy balance of just the atmosphere.

We must have $I_1 = I_2 + I_3$ and so $I_2 = \dfrac{(1-t)(1-\alpha)}{t}\dfrac{S}{4}$.

**b** Now, from the Stefan-Boltzmann law, $I_1 = \sigma T^4$ and so $\dfrac{(1-\alpha)}{t}\dfrac{S}{4} = \sigma T^4$.

We can then determine *t* from this last equation to be 0.63. This is a toy model which shows that with a reasonable percentage of radiation absorbed by the atmosphere, i.e. (1 − 0.63) × 100% = 37%, we get the required earth average temperature without going into the details of the absorption process itself or having to calculate the atmosphere temperature. One can then argue if the 37% of the infrared radiation is absorbed by the atmosphere is in fact realistic etc. in order to criticize the model.

**13 a** Surface heat capacity is the amount of thermal energy required to increase the temperature of a unit area of a body by 1 degree.

**b** Since $C_S = \rho hc$, we have that

$$\frac{C_{\text{water}}}{C_{\text{land}}} = \frac{\rho_{\text{water}} hc_{\text{water}}}{\rho_{\text{land}} hc_{\text{land}}} = \frac{\rho_{\text{water}} c_{\text{water}}}{\rho_{\text{land}} c_{\text{land}}}.$$

The specific heat capacity of rock and sand is about $800\ \text{J kg}^{-1}\,\text{K}^{-1}$ and that of water is $4200\ \text{J kg}^{-1}\,\text{K}^{-1}$. The density of water is $1000\ \text{kg m}^{-3}$ and that of rock and sand is about $5000\ \text{kg m}^{-3}$ so $\frac{C_{\text{water}}}{C_{\text{land}}} = \frac{10^3 \times 4200}{5 \times 10^3 \times 800} \approx 1$.

This is a rough estimate based on the same effective depth of heating. Many references quote a value of 10 for this ratio, which may mean that the effective height cannot be taken to be the same.

**c** With a ratio of 10, we may argue that the same quantity of thermal energy supplied or taken away from water and land will result in a smaller change in temperature in water, i.e. a slower change of climate.

**14** Radiation is a main mechanism to both the atmosphere and to space. In addition there is conduction to the atmosphere, as well as convection.

**15** The albedo of dry land is typically around 0.3 to 0.4, whereas that of forests is lower, at around 0.10 to 0.15. Hence in her models the researcher must increase the albedo.

**16** **a** Dry sub-tropical land has a high albedo, around 0.4, whereas a warm ocean has an albedo of less than 0.2.

**b** Radiation and convection currents are the main mechanisms.

**c** Replacing dry land by water reduces the albedo of the region (see **Q23** and page 448 in *Physics for the IB Diploma*). Reducing the albedo means that less radiation is reflected and more is absorbed, and so an increase in temperature might be expected. The increase in temperature might involve additional evaporation and so more rain.

**17** The answer is given in the hint! Covering vegetation with water will not significantly change the albedo, whereas covering dry land with water would reduce the albedo.

**18** The rate of evaporation from water depends on the temperature of the water and the temperature of the surrounding air. These are both higher in the case of the tropical ocean water and evaporation will be more significant in that case.

**19** **a, b** Evaporation implies that energy has to be supplied to water in order to provide the latent heat required to evaporate the water. This energy comes from the atmosphere and so implies a cooling of the atmosphere.
Increased evaporation means increased formations of cloud cover in the atmosphere and so more reflection of solar radiation from the atmosphere. This implies additional cooling of the atmosphere.

**20** **a** The total reflected intensity is $18+9=27\%$ and so the albedo is 0.27.

**b** The intensity into the planet is 100 % and that sent back into space is $18+9+73=100\%$. The net intensity into the planet is zero and so the temperature must be constant.

**21** There is more evaporation in region **a,** implying that it is both warmer and wet.

The fact that region **b** is warmer is further supported by the somewhat greater conduction which would be expected if the difference in temperature between the atmosphere and the land were larger.

**22** The latent heat flux refers to the energy that is supplied to water in order to evaporate it. Much more evaporation takes place near the equator rather than the poles due to the higher temperatures, and so a reasonable sketch would be a decreasing function as latitude increases.

**23** Following **example Q7** (see page 448 in *Physics for the IB Diploma*) we have that the original average albedo of the area was $0.6\times0.10+0.4\times0.3=0.18$ and the new one is $0.7\times0.10+0.3\times0.3=0.16$ for a reduction in albedo of 0.02.
Hence the expected temperature change is estimated to be $2\ ^{\circ}\mathrm{C}$.

**24** **a** By the greenhouse effect we mean that infrared radiation radiated by the earth's surface gets trapped by the atmosphere and is partly returned back to the earth, increasing the average temperature.

**b** The main greenhouse gases apart from water vapour are carbon dioxide, methane and nitrous oxide. For source and sinks see page 441 in *Physics for the IB Diploma*.

**25** The natural greenhouse effect occurs because there are naturally occurring concentrations of various greenhouse gases in the atmosphere. The enhanced greenhouse effect refers to additional absorption of energy in the atmosphere due to increased concentrations of greenhouse gases due, mainly, to human activity.

**26** The main long-term evidence comes from the ice core samples taken from Antarctica. The samples show a clear correlation between increased carbon dioxide concentrations and increases in average global temperatures.

**27** **a** A transmittance curve is a graph showing the percentage of the intensity of radiation that gets transmitted through matter as a function of the wavelength of the radiation.

**b** The dips at wavelengths of about 2.2 and 4.1 micrometres would be much shallower but that at 7.1 micrometres would be unaffected.

**28** **a** At a very simple level, covering land by water increases the rate of evaporation (hence more clouds and more rain) and also reduces the albedo of the region, implying more radiation being absorbed.

**b** One mechanism is the melting of land-based ice. The water will find its way into the sea and contribute to a rising sea level. Another is that increased global temperatures will result in an expansion of ocean water, also giving rise to increased sea levels.

**29** **a** The energy is $Q = mL = 10^5 \times 330 \times 10^3 = 3.3 \times 10^{10}$ J.

**b** No rise in sea level is expected since the ice is floating in water.

**30** The volume of water is $V = Ah = A \times 1.5 \times 10^3$ m$^3$.
The change in volume is $\Delta V = \gamma V \Delta\theta = 2 \times 10^{-4} \times A \times 1.5 \times 10^3 \times 3 = 9.0 \times 10^{-1} \times A$ m$^3$.
The increase in sea level is then $\dfrac{\Delta V}{A} = 9.0 \times 10^{-1}$ m. This assumes uniform heating of all the water and no flooding of previously dry land.

**31** We are given that $\dfrac{\Delta V}{A} = 6$ m. Assuming an area $A$ of 70% of the Earth's surface covered by water, we find $A = 0.70 \times 4\pi R^2 = 0.70 \times 4\pi \times (6400)^2 = 3.6 \times 10^8$ km$^2$.
This means that $\Delta V = 6 \times 3.6 \times 10^8 = 2.2 \times 10^9$ km$^3$.

**32** There are two main effects to consider. One is that the trees in rain forests are reduced, and the second is what happens to the trees after they are removed. In many instances, the removed trees are burned, and this contributes to greenhouse gases in the atmosphere. The fact that the rain forests are no longer there implies a change in the albedo of the area (from the high albedo of the forest to the low albedo of dry land/asphalt/cement that may replace the forest). This contributes to global warming as the lower albedo implies less energy reflected and more absorbed.

**33** Increased solar activity that implies a larger intensity of radiation received on earth, and increased volcanic activity that implies larger mounts of greenhouse gas concentrations in the atmosphere.

**34** See page 449 in *Physics for the IB Diploma*.

**35** The Kyoto protocol has primarily recommended action to limit carbon emissions.

**36** The Intergovernmental Panel for Climate Change is an international organization that collects and evaluates research on the climate change problems and advises governments and other international organizations.

## Additional problems

**A1 This question is about radiation and the Stefan–Boltzmann law**

A black body at temperature $T_1$ radiates energy towards a body of emissivity $e$ and lower temperature $T_2$. All the energy radiated is either absorbed or reflected, i.e. none is transmitted.

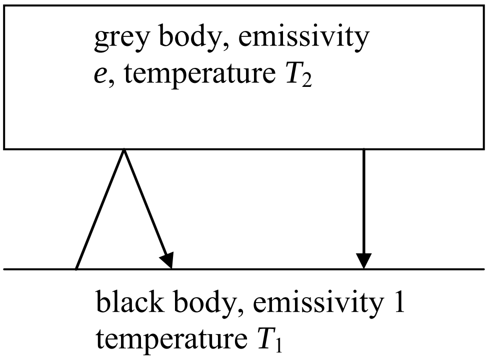
**a** Show that the energy per unit time per unit area lost by the black body is $e\sigma(T_1^4 - T_2^4)$.

**b** Deduce that the intensity absorbed by the surrounding body must also be $e\sigma(T_1^4 - T_2^4)$.

**c** Hence deduce that at equilibrium (i.e. when the temperatures are constant) the two bodies must in fact have the same temperature.

### Solution

**a** The diagram of the situation is the following.




The black body emits an intensity $\sigma T_1^4$ of which a fraction $e\sigma T_1^4$ is absorbed by the surrounding body and hence a fraction $(1-e)\sigma T_1^4$ is returned back to the black body. Since we have a black body, this is all absorbed. In addition, the surrounding body radiates an intensity $e\sigma T_2^4$ all of which is absorbed by the black body. Hence the net radiation intensity **lost** by the black body is

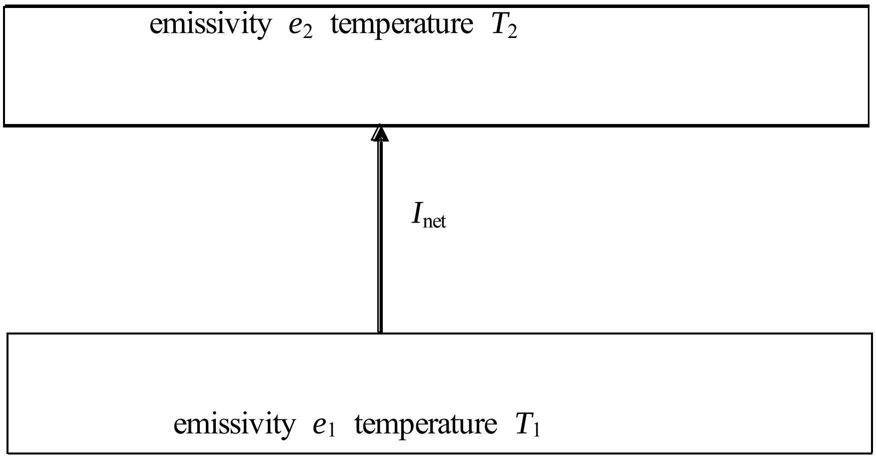
$$\begin{aligned} I_{net} &= \sigma T_1^4 - (1-e)\sigma T_1^4 - e\sigma T_2^4 \\ &= e\sigma(T_1^4 - T_2^4) \end{aligned}$$

**b** The surrounding body absorbs an intensity $e\sigma T_1^4$ and radiates an intensity $e\sigma T_2^4$. Hence the net intensity **absorbed** is $I_{net} = e\sigma(T_1^4 - T_2^4)$. This is obvious because the colder body must receive the energy which the warmer body loses.

**c** If each body is to have a constant temperature, the net intensity must be zero and so $I_{\text{net}} = e\sigma(T_1^4 - T_2^4) = 0$ , implying that $T_1 = T_2$.

**A2 This question is about radiation and the Stefan–Boltzmann law**

Two bodies of emissivities $e_1$ and $e_2$ and temperatures $T_1$ and $T_2$ radiate towards each other. All the radiation is either reflected or absorbed and none is transmitted.




**a** Show that the net intensity of radiation leaving the warmer body is given by

$$I_{net} = \frac{e_1 e_2 \sigma (T_1^4 - T_2^4)}{1-(1-e_1)(1-e_2)}.$$

**b** Deduce that when one of the bodies is a black body this formula reduces to the familiar result of the previous problem.

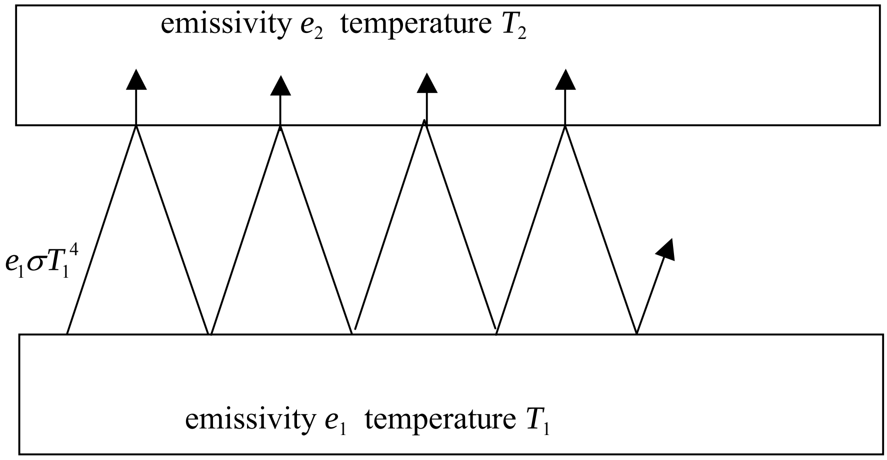
**c** A horizontal shield is placed in between the bodies in order to reduce the net flow of radiation from the warm to the colder body. Assuming that the emissivities of the two bodies and the shield are all the same, $e_1 = e_2 = e$, show that the shield will reduce the flow of radiation by a factor of 2.

(You may want to refer to this problem when you examine the Olbers paradox in the Astrophysics option.)

**Solution**

**a** The diagram shows radiation of intensity $e_1 \sigma T_1^4$ being emitted from the hot body. Some of this radiation is absorbed by the colder body and some is reflected. Of the reflected radiation some is re-absorbed by the hot body and some is reflected and so on. The radiation lost by the hot body is equal to whatever is being absorbed by the colder body, i.e. what is represented by the vertical arrows into the colder body.




Since the emissivity of the colder body is $e_2$, a fraction $e_2$ of the incident intensity will be absorbed. Thus the first arrow representing absorbed radiation has an intensity $e_2(e_1\sigma T_1^4)$. The fraction reflected is $\alpha_2(e_1\sigma T_1^4)$ , where $\alpha_2 = 1-e_2$ is the albedo of the colder body. Of this the reflected radiation from the warmer body will be $\alpha_1\alpha_2(e_1\sigma T_1^4)$ , where $\alpha_1 = 1-e_1$ is the albedo of the warmer body. Of this a fraction $e_2$ will be absorbed, i.e. $e_2(\alpha_1\alpha_2(e_1\sigma T_1^4))$ , and so on. This means that the radiation emitted by the warmer body that is absorbed by the colder body is (we have to sum an infinite geometric series)

$$\begin{aligned}
I_{out} &= e_2e_1\sigma T_1^4 + e_2e_1\alpha_1\alpha_2\sigma T_1^4 + e_2e_1(\alpha_1\alpha_2)^2\sigma T_1^4 + e_2e_1(\alpha_1\alpha_2)^3\sigma T_1^4 + \ldots \\
&= e_2e_1\sigma T_1^4(1+\alpha_1\alpha_2+(\alpha_1\alpha_2)^2+(\alpha_1\alpha_2)^3+\ldots) \\
&= \frac{e_2e_1\sigma T_1^4}{1-\alpha_1\alpha_2} \\
&= \frac{e_2e_1\sigma T_1^4}{1-(1-e_1)(1-e_2)}
\end{aligned}$$

In exactly the same way, the radiation emitted by the colder body that is absorbed by the warmer body is

$$I_{in} = \frac{e_2e_1\sigma T_2^4}{1-(1-e_1)(1-e_2)}$$

Hence the net flow of radiation from the warm to the cold body is

$$\begin{aligned}
I_{net} &= I_{out} - I_{in} \\
&= \frac{e_2e_1\sigma T_1^4}{1-(1-e_1)(1-e_2)} - \frac{e_2e_1\sigma T_2^4}{1-(1-e_1)(1-e_2)} \\
&= \frac{e_2e_1\sigma (T_1^4 - T_2^4)}{1-(1-e_1)(1-e_2)}
\end{aligned}$$

as required.

**b** Assume now that one of the bodies, say the warmer one, is a black body. Then $e_1 = 1$ and the formula becomes $I_{\text{net}} = e_2\sigma(T_1^4 - T_2^4)$, as expected from the previous problem.

**c** With the shield in place, at equilibrium the shield will lose as much radiation as it receives. Let $T$ be the temperature the shield acquires. Then

$$\underbrace{\frac{e^2\sigma T_1^4}{1-(1-e)^2} - \frac{e^2\sigma T^4}{1-(1-e)^2}}_{\text{radiation into shield from warner body}} = \underbrace{\frac{e^2\sigma T^4}{1-(1-e)^2} - \frac{e^2\sigma T_2^4}{1-(1-e)^2}}_{\text{radiation from shield into colder body}}$$

leading to $2T^4 = T_1^4 + T_2^4$.
Hence

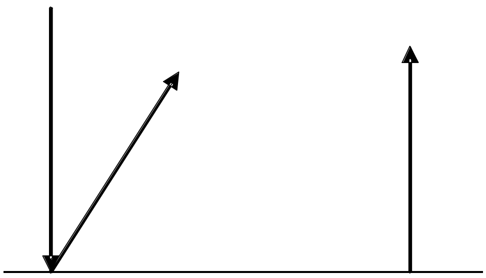
$$\frac{e^2\sigma T^4}{1-(1-e)^2} - \frac{e^2\sigma T_2^4}{1-(1-e)^2} = \frac{1}{2}\frac{e^2\sigma(T_1^4 + T_2^4)}{1-(1-e)^2} - \frac{e^2\sigma T_2^4}{1-(1-e)^2}$$
$$= \frac{1}{2}\frac{e^2\sigma(T_1^4 - T_2^4)}{1-(1-e)^2}$$

i.e. the net intensity transferred is reduced by a factor of 2.

This is of some relevance in the discussion of the Olbers paradox. A possible solution of the paradox that was initially put forward was that gas and dust in space would block the radiation from the distant stars. The problem above makes it clear that the gas and dust (here playing the role of the shield) do not block the radiation – they just reduce it – but the reduction is not sufficient to resolve the paradox.

**A3 Critique of the simple model of the earth energy balance.**

Consider the simple model that was examined in **example Q3** (see page 438 in *Physics for the IB Diploma*).



earth

The diagram shows radiation incident on the earth, part of which is reflected and the rest is absorbed. The diagram also shows the intensity radiated by the earth. Given that the albedo of the earth is $\alpha$ , state the assumptions made in deriving the standard formula $T_e = \sqrt[4]{\frac{S(1-\alpha)}{4\sigma}}$ (the intensity of the radiation at the position of the earth is $S$ )

for the equilibrium temperature of the earth, and comment on the assumptions you state.

**Solution**

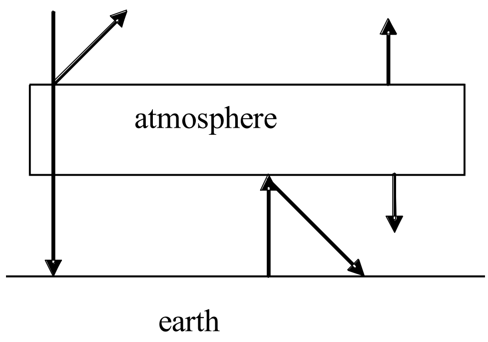
The intensity of the radiation at the position of the earth is $S$ (the solar constant). The earth presents to this incident radiation an area $\pi R^2$ which collects this radiation. Assuming uniform distribution of this radiation over the entire earth surface of area $4\pi R^2$ we see that on the average the incident intensity at an arbitrary point on the earth's surface is $\frac{S}{4}$. Of this radiation $\frac{\alpha S}{4}$ is reflected and $\frac{(1-\alpha)S}{4}$ is absorbed.

The earth radiates an intensity $\sigma T_e^4$. The formula is derived by equating the intensity of radiation into the earth with that out of the earth, i.e. $\sigma T_e^4 = \frac{(1-\alpha)S}{4}$ which is the result.

The big assumption here is that the earth radiates as a black body. This is not a good assumption at all and is in fact inconsistent physically: if the earth were a black body it would not reflect **any** of the radiation incident on it! Other assumptions made in this derivation include the complete neglect of the atmosphere and any interaction between the earth's surface and the atmosphere.

## A4 A model of the earth–atmosphere system

Consider now a more involved model of the energy balance of the earth–atmosphere system. This is an extreme model of the greenhouse effect in which none of the radiation emitted by the earth actually escapes into space.




The diagram shows the incident intensity from the sun, the reflected intensity from the atmosphere, the intensity transmitted through the atmosphere to the earth's surface, the intensity radiated by the surface, and the radiation reflected by the atmosphere and radiated by the atmosphere into both space and towards the earth surface. Treat the earth's surface as a black body, the atmosphere as a body of emissivity $\varepsilon$ and albedo $\alpha$. Take the temperature of the surface to be $T_e$ and that of the atmosphere to be $T_a$. Find an expression for the earth's temperature, stating your assumptions and commenting on them. (Hint: start by stating the value of each of the intensities in the diagram and then write down the conditions that the net intensity into is equal to the net intensity out of each of the bodies **i** earth surface and **ii** atmosphere.

**Solution**

The incident intensity from the sun is $\frac{S}{4}$.

The reflected intensity from the atmosphere is $\frac{\alpha S}{4}$ and that transmitted to the earth's surface is $\frac{(1-\alpha)S}{4}$.

The intensity radiated by the surface is $\sigma T_e^4$ (black body), and each of the intensities radiated by the atmosphere is $\varepsilon\sigma T_a^4$.

The intensity reflected from the atmosphere towards the earth is $(1-\varepsilon)\sigma T_e^4$ since an amount $\varepsilon\sigma T_e^4$ is absorbed.

We now consider the energy balance of the surface: the net intensity into the surface is $\frac{(1-\alpha)S}{4}+\varepsilon\sigma T_a^4+(1-\varepsilon)\sigma T_e^4$ and that radiated out of the surface is $\sigma T_e^4$.

Hence $\frac{(1-\alpha)S}{4}+\varepsilon\sigma T_a^4+(1-\varepsilon)\sigma T_e^4=\sigma T_e^4$.

The energy balance of the atmosphere gives: $2\varepsilon\sigma T_a^4=\varepsilon\sigma T_e^4$ ; hence $2T_a^4=T_e^4$.
Substituting this in the first equation gives:

$$\frac{(1-\alpha)S}{4}+\frac{\varepsilon\sigma T_e^4}{2}+(1-\varepsilon)\sigma T_e^4=\sigma T_e^4$$

$$\frac{(1-\alpha)S}{4}+(1-\frac{\varepsilon}{2})\sigma T_e^4=\sigma T_e^4$$

$$\frac{(1-\alpha)S}{4}=\frac{\varepsilon\sigma T_e^4}{2}$$

$$T_e=\sqrt[4]{\frac{(1-\alpha)S}{2\varepsilon\sigma}}$$

In this case, since no IR radiation from the earth's surface gets transmitted into space it follows that $\alpha=1-\varepsilon$. The model gives a temperature that does not depend on the albedo of the earth!

$$T_e=\sqrt[4]{\frac{S}{2\sigma}}$$

$$T_e=333\text{ K}$$

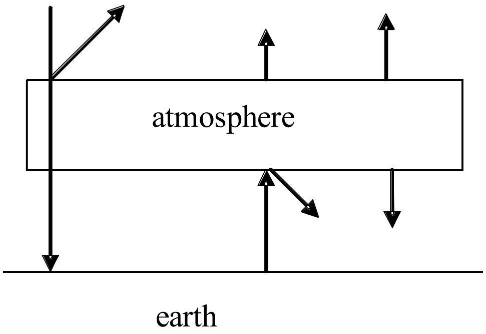
which is a very high temperature! This is not surprising in a model that uses an extreme version of the greenhouse effect.

This model has assumed that: **i** no radiation radiated by the earth actually escapes into space; and **ii** no reflection takes place from the surface, i.e. the earth surface is treated as a black body.

## A5 Yet another model of the earth–atmosphere system

This model will now try to correct the deficiencies of the model in A4 by allowing transmission into space from the earth surface in addition to the features of the model in **A4**.




### Solution

The incident intensity from the sun is $\frac{S}{4}$.

The reflected intensity from the atmosphere is $\frac{\alpha S}{4}$ and that transmitted to the earth's surface is $\frac{(1-\alpha)S}{4}$.

The intensity radiated by the surface is $\sigma T_e^4$ (black body) and each of the intensities radiated by the atmosphere is $\varepsilon\sigma T_a^4$.

The intensity reflected from the atmosphere towards the earth is $\alpha\sigma T_e^4$, an amount $\varepsilon\sigma T_e^4$ is absorbed and an amount $t\sigma T_e^4$ , where $t$ is a transmission coefficient, gets transmitted through the atmosphere and out into space.

We must have that $1 = \alpha + \varepsilon + t$ (by conservation of energy).

We now consider the energy balance of the surface: the net intensity into the surface is $\frac{(1-\alpha)S}{4} + \varepsilon\sigma T_a^4 + \alpha\sigma T_e^4$ and that radiated out of the surface is $\sigma T_e^4$.

Hence $\frac{(1-\alpha)S}{4} + \varepsilon\sigma T_a^4 + \alpha\sigma T_e^4 = \sigma T_e^4$.

The energy balance of the atmosphere gives:

$$2\varepsilon\sigma T_a^4 + t\sigma T_e^4 + \alpha\sigma T_e^4 = \sigma T_e^4$$
$$\Rightarrow 2\varepsilon\sigma T_a^4 = \varepsilon\sigma T_e^4$$

hence $2T_a^4 = T_e^4$ just as before.

Substituting this in the first equation gives:

$$\frac{(1-\alpha)S}{4}+\frac{\varepsilon\sigma T_e^{\,4}}{2}+\alpha\sigma T_e^{\,4}=\sigma T_e^{\,4}$$

$$\frac{(1-\alpha)S}{4}=\sigma T_e^{\,4}(1-\frac{\varepsilon}{2}-\alpha)$$

$$\frac{(1-\alpha)S}{4}=\frac{(2-\varepsilon-2\alpha)\sigma T_e^{\,4}}{2}$$

$$T_e=\sqrt[4]{\frac{(1-\alpha)S}{2(2-\varepsilon-2\alpha)\sigma}}$$

Choosing $\alpha=0.30$ , $t=0.06$ and so $\varepsilon=0.64$ gives an earth's surface temperature of

$$T_e=\sqrt[4]{\frac{(1-0.30)\times 1400}{2(2-0.64-2\times 0.30)\times 5.67\times 10^{-8}}}$$

$$T_e=327\text{ K}$$

This is only slightly better than the model in **A4**. It is clear that it is not possible to obtain realistic answers with simple models of this kind that neglect other interactions between the surface and the atmosphere.